Introducing the Kalman filter and application

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Abstract — Kalman Filter (KF) is a conventional algorithm for estimation and prediction especially when data has a lot of noise. KF is used for linear transition functions where as under non-linear transition, Extended Kalman Filter (EKF) is used. The EKF is also considered to be the de-facto standard. There are many articles about EKF however very few of them show working of the algorithm. The purpose of this paper is to explain how to apply EKF on a simple application, explaining model creation, substitution, simulation and illustration of adjusting the accuracy of the model.

Index Tems — Kalman Filter, Extended Kalman Filter, Kalman filter application.

1 Introduction

Kalman filter is a recursive algorithm used for estimating a dynamic system which lacks data. The cause of lack of data is noise or environment, an example of such systems includes: autonomous and assisted navigation system. Kalman filter uses prior knowledge to predict many kind states of system. They are past, present as well as the future state of the system. The advantage of KF is that the data is updated in each and every iteration so that we do not need much memory to keep all data of the system for prediction.

The Kalman filter is a data integration algorithm that combines the noise sensor measurements flow over time with a model-based prediction. The algorithm can be applied into systems which use Global Positioning System signal receiver, gyroscope sensor or any other inputs related to determining system status. The status can be the position of an object, altitude of an aircraft, velocity of a missile or some other object of interest.

This paper is arranged with seven sections. We will present Kalman filter and Extended Kalman filter in section 2. Section 3 discusses Applying Extended Kalman Filter. And in section 4, Conclussion is shown. Acknowledgment, and References are presented at the end of the paper, respectively.

2 Processing

2.1 Kalman Filter

The Kalman filter solves a common problem of trying to estimate the state $x \in R^n$ of a discrete-time control process governed by the linear probability difference equation:

$$x_t = Cx_{t-1} + Du_{t-1} + w_{t-1} \ , \tag{1}$$

where $y \in R^m$ is a measurement

$$y_t = Kx_t + v_t \ . \tag{2}$$

At the beginning of some initial state estimate, $\hat{x}_0$, and initial state error covariance matrices, A_0, the predictor modification format is applied recursively at each time, for example, using a loop. First, the state vector is predicted from the state dynamic equation using:

$$\hat{x}_t^- = C\hat{x}_{t-1} + Du_{t-1} \ , \tag{3}$$

where $\hat{x}_t^-$ is the predicted state vector

$\hat{x}_{t-1}$ is the previously estimated state vector

u is the input vector

C and D are matrices used to define the dynamic systems

The index sign (-) indicates that the variable is predicted using the previous estimates and the exponent symbol (^) denotes the variable as estimates. This is the prediction of the state using the system model projected forward one step in time. In the second step, the state error covariance matrix has to also be predicted by using the equation below:

$$A_t^- = CA_{t-1}C^T + M \ , \tag{4}$$

where represents the predicted state error covariance matrix is represented by A_t^-, the value A_{t-1} is the previous estimated state error covariance matrix. The process noise covariance matrix is denoted by M.

The Kalman gain matrix, G_t, is calculated by:

$$G_t = A_t^- K^T \left(K A_t^- K^T + N \right)^{-1} \ , \tag{5}$$

where matrix necessary is defined by K, the measurement noise covariance is denoted by N.

In Equation 5, we can see the influence of the sensor measurement noise matrix N on the Kalman gain. The state vector is then updated by scaling the "innovation," which is the difference between the measurement and the predicted output, corresponding to y_t and $K\hat{x}_t^-$, respectively.

$$\hat{x}_t = \hat{x}_t^- + G_t \left(y_t - K\hat{x}_t^- \right) \ . \tag{6}$$

Similarly, the state error covariance is updated by:

$$A_t = \left(I - G_t K \right) A_t^- \ , \tag{7}$$

where I is an identity matrix [1].

As we can see, it can be seen that the Kalman filtering algorithm attempts to converge into correct estimations, even if the Gaussian noise parameters are incorrectly estimated. KF results from continuously updating the estimation data based on prior knowledge in each and every iteration of prediction and filtering [2]. The changing in each iteration is derived and interpreted in the term of Gaussian probability density functions. The innovation process of KF associated with the filter, that represents the novel prediction of information in noisy environment conveyed to the state estimate by using data in the last system measurement. The optimal non-linear filters propagate these non-Gaussian functions and calculate their average indicating a high computational burden. Another approach to solving this issue, in the frame of linear filters, is the Extended Kalman filter (EKF).

2.2 Extended Kalman filter

The difference between the Extended Kalman Filter and Kalman Filter is that the state and/or of the output equations have nonlinear functions. Therefore, instead of considering systems of the form (8) and (9), the EKF can consider a more general set of nonlinear equations:

$$x_t = f\left(x_{t-1}, u_{t-1}, w_{t-1}\right) , \quad (8)$$

$$y_t = k\left(x_t, v_t\right) , \quad (9)$$

where the random variable is denoted ~~be~~ by w_{t-1} and measurement noise is denoted by v_t The vector-valued nonlinear state transition function is f, k is the vector-valued, output function or nonlinear observation.

With the aim of predicting a combination non-linear process with measurement relationships, we have some equations as follows:

$$x_t \approx \tilde{x}_t + C\left(x_{t-1} - \hat{x}_{t-1}\right) + W w_{t-1} , \quad (10)$$

$$y_t \approx \tilde{y}_t + K\left(x_t - \tilde{x}_t\right) + V v_t , \quad (11)$$

where the actual state and measurement vectors are x_t and y_t. The value of $\tilde{x}_t$ is the approximate state and measurement vectors are $\tilde{y}_t$. $\hat{x}_t$ is a posterior estimate value of the state at step t. Jacobian matrices are computed which the aim to handle the nonlinear functions:

$$C_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}\left(\hat{x}_{t-1}, u_{t-1}, 0\right) , \quad (12)$$

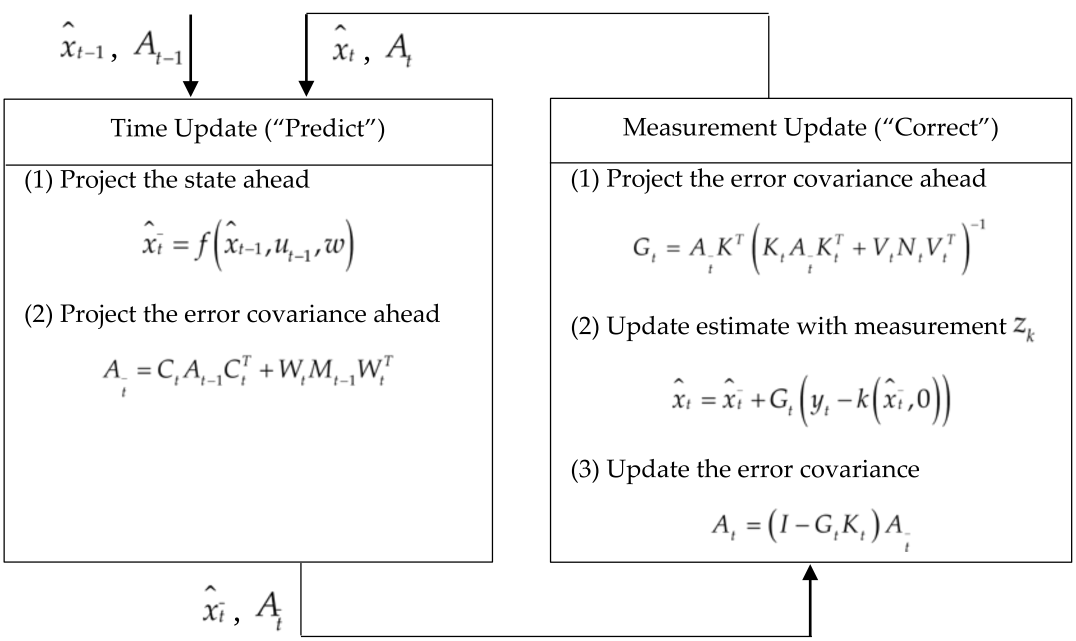
$$W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}\left(\hat{x}_{t-1}, u_{t-1}, 0\right) , \quad (13)$$

$$K_{[i,j]} = \frac{\partial k_{[i,j]}}{\partial x_{[j]}}\left(\tilde{x}_t, 0\right) , \quad (14)$$

$$V_{[i,j]} = \frac{\partial k_{[i]}}{\partial v_{[j]}}\left(\tilde{x}_t, 0\right) , \quad (15)$$

Equation (12), (13), (14), (15) are the Jacobian matrices of partial derivatives of f, k with respect to x, w, v; respectively.

The algorithmic loop of recursive filter is summarised in the diagram of Fig. 1:



x = State model C = State transition matrix D = Control matrix u = Control matrix w = Gaussian white noise	A = State variance matrix K = Measurement matrix G = Kalman gain N = Measuremant variance matrix y = Measurement variables	x_{t-1} = Current state $\hat{x}$ = Estimate state x_t = Future state I = Identity maxtrix $A_{\bar{t}}$ = Previous state variance

Fig. 1: A process of the Extended Kalman Filter [1]

3 Applying Extended Kalman Filter

For applying Kalman Filter, knowledge of mathematics, physics is most important which is required for model creation. In this paper, a simple example problem is offered. We examine tracking an object in a plane. Firstly, we need to model the movement of this object. We can't model accurately the object's movement, but we can have an acceptable approximation model of the object movement. Assuming that the motion on the x-axis is uncorrelated to the motion on the y-axis and the motion on both of the x-axis and y-axis are uncorrelated to the angular rotation around the z-axis, we can write the following discrete equations that describe the object's movements as shown below:

$$x(t+1) = x(t) + T_s v_x(t) + \frac{T_s^2}{2} a_x(t),$$
$$y(t+1) = y(t) + T_s v_y(t) + \frac{T_s^2}{2} a_y(t),$$
$$\theta(t+1) = \theta(t) + T_s \omega(t) + \frac{T_s^2}{2} \alpha(t),$$
$$v_x(t+1) = v_x(t) + T_s a_x(t),$$
$$v_y(t+1) = v_y(t) + T_s a_y(t),$$
$$\omega(t+1) = \omega(t) + T_s \alpha(t).$$

And we can write to them as a state-space model as follows:

$$X(t+1) = \begin{bmatrix} x(t+1) \\ y(t+1) \\ \theta(t+1) \\ v_x(t+1) \\ v_y(t+1) \\ \omega(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 & T_s & 0 \\ 0 & 0 & 1 & 0 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \\ v_x(t) \\ v_y(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{T_s^2}{2} & 0 & 0 \\ 0 & \frac{T_s^2}{2} & 0 \\ 0 & 0 & \frac{T_s^2}{2} \\ T_s & 0 & 0 \\ 0 & T_s & 0 \\ 0 & 0 & T_s \end{bmatrix} \begin{bmatrix} a_x(t) \\ a_y(t) \\ \alpha(t) \end{bmatrix}.$$

And it can be written as:

$$X(t+1) = CX(t) + Du(t) + w.$$

Assuming w is a Gaussian distribution noise with a mean 0 and a variance M:

$$M = DM_m D^T = D \begin{bmatrix} \sigma_{a_x}^2 & 0 & 0 \\ 0 & \sigma_{a_y}^2 & 0 \\ 0 & 0 & \sigma_\alpha^2 \end{bmatrix} D^T.$$

In practice, the value M is unknown, and we will have to estimate it.

We can write a measurement system of the model as follows:

$$Y(t+1) = KX(t+1) + v,$$

where $K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ and v is the measurement noises that are introduced by the means of measurements. We assume the measurements noises as a Gaussian distribution with a mean of 0 and a variance N:

$$N = KK^T V = KK^T \begin{bmatrix} \sigma_{y_x}^2 & 0 & 0 \\ 0 & \sigma_{y_y}^2 & 0 \\ 0 & 0 & \sigma_{y_\theta}^2 \end{bmatrix}.$$

Unlike other types of filters, the Kalman filter requires to give it the exact initial state of the object and the correct initial covariance. Therefore, if you can't provide a correct initial position and covariance matrix for the Kalman filter, it will fail.

In Fig. 2, we see that the filter estimates the movement of the object on the x-axis, y-axis and θ-angle quite well. The blue line is the actual path of the object and the red line is the estimated path of the filter almost identical.

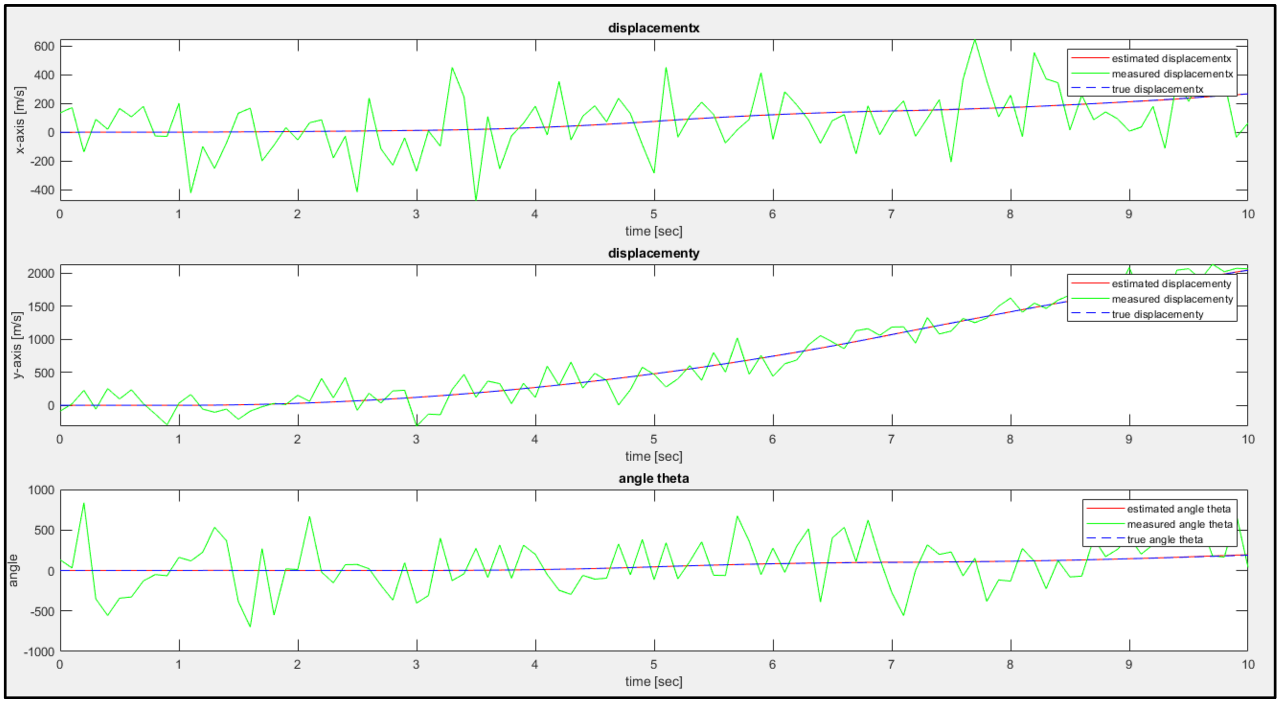


Fig. 2: Tracking objects using the Kalman filter when M = 0.01

4 Conclusion

In this article, we change the values of Q and analyze its ffect. The results of our simulation (Fig. 2, Fig. 3 and Fig. 4) depict the effect of varying the values of M.

In Fig. 2, M is assigned the value 0.01, it is suitable for successful model.

In Fig. 3, M is given the value of 0.5.

In Fig. 4, M is assigned the value 1, quite different from 0, depicting a no trust state on the measured variable.

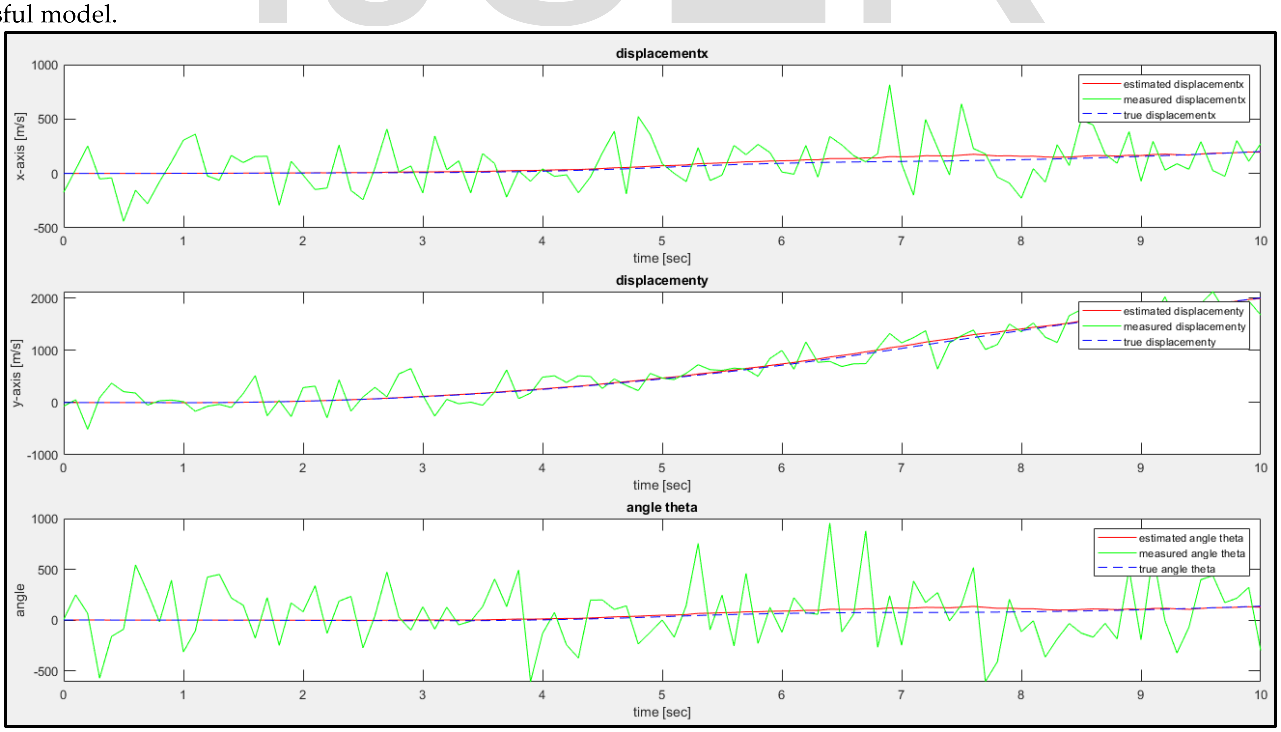


Fig. 3: Tracking objects using the Kalman filter when M = 0.5

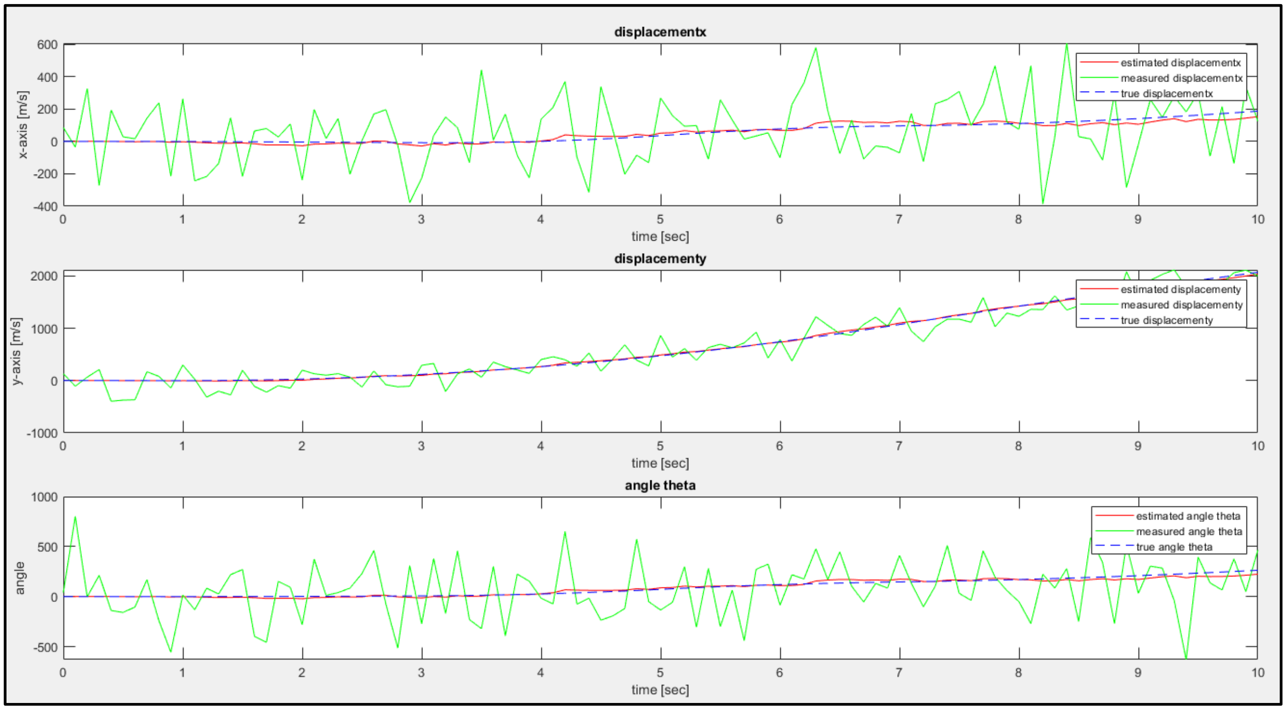


Fig. 4: Tracking objects using the Kalman filter when M = 1

Therefore, the estimated value is based on the results obtained from the transition function in the state model. Defining the noise parameters show that every parameter in EKF is important for adjusting an accuracy level of the model.

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